Global Versus Local Symmetries

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Introduction

Symmetry has been a cornerstone of human thought and aesthetics since ancient times in various civilizations. While the ancient interpretation of symmetry encompassed the idea of equal arrangement and proportion, the modern understanding is limited to the set of transformations that leave the object invariant. [1]

Thesis

We investigate the concept of partial (local) symmetry, which may be viewed as a sort of return to the original meaning of the term symmetry, stressing the importance of proportionality but capturing the current meaning of symmetry as well. Moreover, we investigate its significance in various disciplines, such as neuroaesthetics and mathematics. Furthermore, we argue that the concept of local (partial) symmetry, as opposed to global (total) symmetry, is more natural, more general, and better describes natural phenomena and symmetries in abstract structures.

Argument

We argue that the concept of symmetry is insufficient to capture the complexity of the world because 1.) it is very restrictive to just slight asymmetries, and 2.) symmetries hold too much redundant information and are therefore extremely simple to generate complexity, while the concept of partial symmetry provides a more accurate representation. [2] 3) The concept of partial symmetry better addresses proportion, emphasizing both the whole and its parts, akin to the master tilers' approach when tiling objects.

Methods

Building on the work of [2] and [3], we develop the relevant part of the theory of inverse semigroups axiomatically and logically. Then, we synthesize perspectives from various disciplines to analyze how partial symmetry is treated in different fields and identify unifying principles in these approaches. Examples and images are employed to establish a comprehensive and consistent understanding of the (partial) symmetry concept.

Results and Conclusion

With the synthesis, we find that:

1. The concepts of partial symmetry and symmetry should not be viewed in opposition but as very close concepts.
2. The concept of symmetry is insufficient to capture the complexity of nature and cognition (it fails to give any information when there is just a slight asymmetry).
3. The concept of partial symmetry is a suitable candidate to capture the complexity of nature and cognition.

We think that by embracing the concept of local (partial) symmetry, researchers across disciplines can gain a more nuanced representation of the structure of objects in both natural and abstract contexts.

References

[1] G. Hon and B. R. Goldstein, From Summetria to Symmetry: The Making of a Revolutionary Scientific Concept, Springer, 2008.

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[3] M. V. Lawson, Inverse semigroups, the theory of partial symmetries, World Scientific, 1998.